Constant Weighted Maximin Share Approximations for Chores

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We study the fair allocation of indivisible chores among agents with asymmetric weights. Among the various fairness notions, weighted maximin share (WMMS) stands out as particularly compelling. Despite its appeal, the existence of a constant-factor approximation for WMMS has remained an important open problem in weighted fair division [Aziz et al., 2022, Suksompong, 2025]. Prior to our work, the best known approximation ratio was $O(\log n)$, where n is the number of agents. In this paper, we make significant progress by presenting the first constant-factor approximation algorithm for WMMS. Our main contributions are as follows:

- We design the first algorithm that guarantees a 12-approximate WMMS allocation, substantially improving upon the previous $O(\log n)$ upper bound. Our approach introduces a novel analytical framework based on canonical instance reductions, agent delegation, and proxy cost functions to effectively bound agents' costs. Additionally, we provide a polynomial-time implementation for any approximate WMMS algorithm, incurring a factor of 2 loss in the approximation ratio.
- We present an improved worst-case lower bound, showing that no algorithm can achieve better than 2-approximate WMMS, thereby strengthening the previous best lower bound of 1.366. We further construct a general hard instance, which provides lower bounds for an arbitrary number of agents.
- Beyond worst-case bounds, we precisely characterize the optimal approximation ratio curve for every possible weight distribution in the two-agent case. Notably, our results imply that a WMMS allocation may not exist for any two agents with different weights, in sharp contrast to the symmetric case where an MMS allocation always exists.

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